An overview of power-law creep in polycrystalline 
β-titanium
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Abstract

A constitutive law for power-law creep of BCC β-Ti is developed, based on experimental data from eight independent studies. The present compilation adds more than twice as many data points as previous analyses, covers nine orders of magnitude in strain rate from $10^{-7}$ to $10^{2}$ s$^{-1}$, and incorporates recent data for the shear modulus of β-Ti. © 2001 Published by Elsevier Science Ltd. on behalf of Acta Materialia Inc.

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Introduction

An early creep study, by Bühler and Wagener [1], on the high-temperature BCC β-phase of titanium was used by Ashby and coworkers [2,3] to construct the deformation mechanism map for titanium. However, the data of Bühler and Wagener were limited to high creep rates in the range 0.1–120 s$^{-1}$, and estimates of creep rates at lower stresses relied upon extrapolation of the power-law (with a stress exponent of \textasciitilde 4.3) across many orders of magnitude. Twenty years later, Oikawa et al. [4] substantially extended the range of measured strain rates to values as low as $5 \times 10^{-5}$ s$^{-1}$, and showed that a consistent power-law could be fitted to their data and the earlier work of Bühler and Wagener, with an activation energy for creep of $Q = 153$ kJ/mol and a stress exponent of $n = 4.1$.

Since the work by Oikawa et al. [4], there have been several additional investigations of creep in β-Ti, for its implications in the deformation and failure of two-phase $\alpha/\beta$

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titanium alloys [5–7], as well as transformation superplasticity of titanium and titanium composites [8,9]. Additionally, one earlier study by Lunsford and Grant [10] has not been previously used in development of the constitutive creep power-law for β-Ti. In this note, we collect and compare these many independent sets of data, and develop a constitutive equation for power-law creep of β-Ti that spans nine orders of magnitude in strain rate between $10^{-7}$ and $10^2$ s$^{-1}$.

**Discussion**

Eight creep studies on polycrystalline β-Ti are summarized in Table 1, where the range of applied stresses, $\sigma$, temperatures, $T$, and steady-state or minimum creep strain-rates, $\dot{\varepsilon}$, are presented along with the concentrations of the major impurities. In all but one of the studies, the material had purity in the range 99.36–99.89 wt.%. The exception is the iodide titanium (initially of 99.92% purity) of Lunsford and Grant [10], who performed their creep testing in air, and observed substantial absorption of airborne elements. The purity of their specimens during creep testing is therefore unknown, but certainly below the value given in Table 1. Grain growth is extremely rapid in β-Ti, since the $\alpha/\beta$ transformation temperature at 882 °C corresponds to a homologous temperature of 0.69; it is therefore likely that all of the data in these studies was collected at large grain sizes, which we estimate to be >200 μm. Furthermore, as we describe subsequently, all of these studies observed power-law, dislocation creep, which is generally grain-size independent [3].

Fig. 1 shows the creep rate of β-Ti as a function of the applied stress, on double-logarithmic scales, from all of the studies in Table 1. Data points from the more extensive investigations of Bühler and Wagener [1], Oikawa et al. [4], and Senkov and Jonas [6] are plotted with different symbols for the various test temperatures, to illustrate the impact of this variable on the creep rate. In general, the creep strain rate follows a power-law in stress, with a stress exponent close to the value of $n = 4.3$ proposed by Ashby and co-workers [2,3], based only on the high strain-rate data of Bühler and Wagener [1].

Power-law creep with a stress exponent in the range $n = 3–10$ is usually attributed to dislocation climb-limited deformation, which can be described by a generalized equation of the form:

$$\dot{\varepsilon} = A \frac{D_v(T) \cdot \mu(T) \cdot b}{k \cdot T} \left( \frac{\sigma}{\mu(T)} \right)^n$$  \hspace{1cm} (1)

where $A$ is the temperature-independent Dorn constant, $b = 0.286$ nm is the Burger’s vector for β-Ti [3], $k$ is the Boltzmann constant, $D_v$ is the coefficient of volume diffusion, and $\mu$ is the shear modulus. The main temperature dependence for creep is through the Arrhenius term of the diffusivity:
Table 1
Summary of creep studies on unalloyed β-Ti

<table>
<thead>
<tr>
<th>Investigation</th>
<th>Year</th>
<th>$T$ (°C)</th>
<th>$\sigma$ (MPa)</th>
<th>$\dot{\epsilon}$ (s$^{-1}$)</th>
<th>Composition (wt.%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Ti</td>
</tr>
<tr>
<td>Lunsford and Grant* [10]</td>
<td>1957</td>
<td>927–1087</td>
<td>2.2–5.0</td>
<td>$8.3 \times 10^{-1}$–$6.7 \times 10^{-1}$</td>
<td>99.92</td>
</tr>
<tr>
<td>Bühler and Wagener [1]</td>
<td>1965</td>
<td>900–1300</td>
<td>6.0–51.1</td>
<td>0.12–120</td>
<td>99.89</td>
</tr>
<tr>
<td>Oikawa et al. [4,14]</td>
<td>1985</td>
<td>907–1027</td>
<td>1.6–8.8</td>
<td>$5.5 \times 10^{-3}$–$5.5 \times 10^{-2}$</td>
<td>99.80</td>
</tr>
<tr>
<td>Braga et al. [5,15]</td>
<td>1993</td>
<td>900–950</td>
<td>13.5–39.0</td>
<td>0.1–3.0</td>
<td>99.71</td>
</tr>
<tr>
<td>Dunand and Bedell [8]</td>
<td>1996</td>
<td>1000</td>
<td>0.5–2.0</td>
<td>$2.0 \times 10^{-3}$–$1.2 \times 10^{-4}$</td>
<td>&gt;99.50</td>
</tr>
<tr>
<td>Senkov and Jonas [6]</td>
<td>1996</td>
<td>920–1030</td>
<td>3.4–23.9</td>
<td>$9.9 \times 10^{-4}$–1.0</td>
<td>99.36</td>
</tr>
<tr>
<td>Ray et al. [7]</td>
<td>1997</td>
<td>900, 950</td>
<td>45.0–100.0</td>
<td>1.0–20.0</td>
<td>99.81</td>
</tr>
<tr>
<td>Schuh et al. [9]</td>
<td>1999</td>
<td>970</td>
<td>0.7–2.3</td>
<td>$5.0 \times 10^{-7}$–$1.6 \times 10^{-4}$</td>
<td>&gt;99.50</td>
</tr>
</tbody>
</table>

* Listed composition is prior to creep testing in air.
Fig. 1. Creep rate of β-Ti as a function of applied stress. Dashed lines show approximate temperatures for the nearby data points.

\[
D_v = D_0 \cdot \exp\left(-\frac{Q}{k \cdot T}\right)
\]

(2)

where \( D_0 \) is a constant and \( Q \) is the activation energy for volume diffusion. The shear modulus is also, in general, temperature-dependent. For the case of β-Ti, no temperature-dependent measurements of the shear modulus were available prior to 1996, so Ashby and coworkers [2,3] used a standard temperature-dependence of the form:
\[ \mu = \mu_0 \cdot \left(1 + \alpha \cdot \frac{T - 300}{T_m} \right) \]  

(3)

where \( T_m = 1933 \text{ K} \) is the melting temperature and \( \mu_0 = 20.5 \text{ GPa} \) was fixed to a single crystal measurement at 1000 °C by Fisher and Dever [11]. The temperature-dependency parameter \( \alpha = -0.5 \) was assumed, and falls in the middle of the range of values for other BCC metals (\( \alpha = -1.2 \to -0 \)) [3]. Using Eqs. (3) and (2) with \( D_0 = 1.9 \times 10^{-7} \text{ m}^2 \text{ s}^{-1} \) and \( Q = 153 \text{ kJ/mol} \) (from Refs. [3,12]), Oikawa et al. [4] showed that the temperature dependencies of their creep data and those of Bühler and Wagener [1] were adequately accounted for.

Although the assumed form of Eq. (3) is very weakly temperature-dependent, the recent acoustic measurements of Senkov et al. [13] have found that the shear modulus \( \mu \approx 17.5 \text{ GPa} \) is essentially temperature-independent for β-Ti over the range \( T = 925-1100 \text{ °C} \). In contrast, Eq. (3) predicts a change from \( \mu = 15.8 \) to 14.8 GPa over this range.

Fig. 2 shows the creep data from Fig. 1, normalized for temperature according to Eq. (1). For comparison, the data are plotted twice on separate axes, using the assumed temperature-dependent shear modulus of Ashby et al. [2,3] (Eq. (3)), as well as the recently-measured temperature-independent shear modulus of Senkov et al. [13]. The latter modulus is more successful in normalizing the data, as observed in Fig. 2 by the smaller amount of scatter. Because of this result, and because the temperature dependence of Eq. (3) was assumed, we suggest that a temperature-independent modulus, \( \mu = 17.5 \text{ GPa} \), based on the measurements of Senkov et al. [13], be used in Eq. (1) to predict creep rates of β-Ti. We note that a similar temperature-independent shear modulus has been used by Frost and Ashby [3] for BCC niobium. Additionally, we find that varying the creep activation energy within a physically-reasonable range of \( Q = 153 \pm 20 \text{ kJ/mol} \) does not significantly improve the normalization of the data in Fig. 2. Finally, it is worthwhile to note that the creep data which fall below the trend-line in Fig. 2 tend to be those of lower-purity titanium, while those above the line are of higher purity (Table 1), as expected if solid-solution hardening is slowing the creep rate of titanium.

Except for the data of Ray et al. [7], all of the temperature-compensated data fall very close to a single power-law relation in Fig. 2. Neglecting the data of those authors [7], the sets of data from all seven independent investigations can be described by Eq. (1) with best-fit values of \( A = 1.3 \times 10^5 \) and \( n = 4.2 \), as shown by the solid line in Fig. 2. This relationship adequately describes the creep behavior of β-Ti over the nine decades of strain rate that have been investigated to date. Finally, we note that the new creep parameters described above are very close to those originally proposed by Ashby and coworkers [2,3] (\( A = 1.0 \times 10^5 \) and \( n = 4.3 \)) and by Oikawa et al. [4] (\( A = 8.0 \times 10^4 \) and \( n = 4.1 \)). This agreement is noteworthy considering that the new values of \( A \) and \( n \) incorporate six new independent data sets (more than twice as many data points), extend the range of strain rates by two additional orders of magnitude, and use an updated form for the temperature dependence of the shear modulus.
Fig. 2. Temperature-compensated creep rate vs. modulus-compensated stress, for the data sets shown in Fig. 1. Upper data set uses the shear modulus of Eq. (3), and lower set uses a temperature-independent modulus. Best fit excludes data of Ray et al. [7].

Conclusions

Eight independent studies on the creep of β-Ti have been assembled and compared. The creep of β-Ti is found to follow a typical power-law over strain rates from $10^{-7}$ to $10^2$ s$^{-1}$, stresses from 0.5 to 50 MPa, and temperatures from 900 to 1300 °C. When new data on the shear modulus of β-Ti are used as input to the power-law, the data are consistently described with new values for the Dorn constant $A = 1.3 \times 10^5$ and the power-law stress exponent $n = 4.2$. 
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