Monkman-Grant Analysis of Creep Fracture in Dispersion-Strengthened and Particulate-Reinforced Aluminum

D.C. DUNAND, B.Q. HAN, and A.M. JANSEN

The tensile creep fracture properties of coarse- and fine-grained dispersion-strengthened-cast aluminum (DSC-Al) with 25 vol pct of submicron alumina dispersoids are presented for temperatures between 335 °C and 500 °C and stresses between 30 and 100 MPa. The primary, secondary, and tertiary creep strains are analyzed in terms of the minimum creep rate, applied stress, and temperature. Good agreement with the original and the modified Monkman–Grant relationships is found for the failure time of DSC-Al and other aluminum materials reinforced with dispersoids or particulates. The origin of the Monkman–Grant relationships for these materials is discussed in terms of stress exponents, specific interfacial areas, and ratio of secondary strain to failure strain.

I. INTRODUCTION

The Monkman–Grant relationship(1) relates the failure time \( t_f \) to the minimum (or secondary) creep rate \( \dot{\varepsilon}_{\text{min}} \) as

\[
t_f \cdot \dot{\varepsilon}_{\text{min}} = C \tag{1}
\]

where \( m \) and \( C \) are constants. For the metals and alloys originally evaluated by Monkman and Grant, the exponent \( m \) had values between 0.8 and 0.95, while the constant \( C \) varied between 3 and 20 (with \( t_f \) and \( \dot{\varepsilon}_{\text{min}} \) expressed in units of hours and \( \text{pct} \cdot \text{hr}^{-1} \), respectively) depending on the material system. A better fit for many materials can be obtained by introducing the failure strain \( \varepsilon_f \) into Eq. [1].(2) The modified Monkman–Grant relationship then takes the form

\[
t_f \cdot \dot{\varepsilon}_{\text{min}} = \varepsilon_f \tag{2}
\]

where \( m' \) is close to unity and \( C \) is a temperature-independent constant. The modified Monkman–Grant relationship with \( m' = 1 \) reflects that the mean creep rate \( \dot{\varepsilon}_f / t_f \) is linearly related to the minimum creep rate \( \dot{\varepsilon}_{\text{min}} \).

Another correlation has been observed to exist for many materials between the applied stress \( \sigma \) and the Larson–Miller parameter \( P \):

\[
P = T \cdot (K + \log t_f) \tag{3}
\]

where \( K \) is about 20 when the failure time \( t_f \) and the temperature \( T \) are expressed in units of hours and Kelvin, respectively.(3) Similarly, the applied stress can be correlated with the Orr–Sherby–Dorn parameter \( \theta \):

\[
\theta = Q/(2.303 \cdot R \cdot T) - \log t_f \tag{4}
\]

where \( Q \) is the activation energy and \( R \) is the gas constant.

If the minimum strain rate is related to the stress through an exponential law, the Larson–Miller parameter (Eq. [3]) can be derived from the Monkman–Grant relationship (Eq. [1]) with \( m = 1 \). Conversely, if minimum strain rate and stress are related through a power law, the Orr–Sherby–Dorn parameter (Eq. [4]) can be derived from the Monkman–Grant relationship with \( m = 1 \).(4)

The technological significance of the Monkman–Grant relationship is that once the constants \( C \) and \( m \) have been determined from a limited number of creep tests on a given material, the relationship can be used (1) to estimate the time to rupture of a long-time test as soon as the minimum creep rate is reached and (2) to check the reliability of individual creep-rupture tests. Similarly, once the constant \( K \) or \( Q \) is known for the Larson–Miller or Orr–Sherby–Dorn parameters, the failure time can be predicted based on the testing temperature and stress. However, because plots of stress vs Larson–Miller or Orr–Sherby–Dorn parameter often exhibit a pronounced curvature, these relationships can be used reliably for extrapolation of failure time only over a limited range of stress and temperature.

The scientific significance of the Monkman–Grant relationship is that it is an important guide to the identification of creep rupture mechanisms. While the Monkman–Grant relationship was originally proposed as a phenomenological correlation, it can be theoretically derived by considering individual high-temperature fracture mechanisms. Raj and Ashby(5) considered void nucleation and cavity growth for several cases. For continuous nucleation on grain boundaries exhibiting grain boundary sliding, the failure time is dependent on creep rate, grain size, particle size, particle volume, and distribution of particles. These authors also considered the case where the nucleation time is an important component of the failure time. Chuan et al.(6) examined void growth from an initial radius to the coalescent radius and considered the failure time to be controlled by both surface and grain boundary diffusion at the early stage and only by surface diffusion at the later stage. Assuming that cavity nucleation throughout creep is rapid, intergranular fracture occurs when cavities start to coalesce, and cavity growth is controlled by coupled diffusion and power-law creep, Edward and Ashby(7) developed a model for void growth: at low temperatures and high strain rates, the product of failure time and minimum creep rate is con-
controlled by power-law creep and increases as the stress exponent decreases, while at high temperatures and low strain rates, the product of failure time and creep rate is controlled by diffusion creep. Needlemen and Rice\textsuperscript{[10]} discussed the preceding model and concluded that the product of failure time and creep rate should be determined from considerations of cavity nucleation and growth. Cocks and Ashby\textsuperscript{[11]} and Cocks\textsuperscript{[12]} further analyzed the nucleation and growth of voids in a material containing grain-boundary particles and concluded that the product of failure time and creep rate is determined by grain-boundary diffusion at high stresses, while at low stress, it is determined by the ability of the matrix material to deform plastically.

The creep behavior of dispersion-strengthened-cast aluminum (DSC-Al) containing high volume fractions of dispersoids was investigated between 335 °C and 500 °C in a previous publication.\textsuperscript{[13]} Cavitation was found to be the dominant deformation mechanism in the secondary creep regime for both large- and fine-grained DSC-Al, resulting in tensile creep rates significantly higher than in compression, where dislocation creep was dominant. While a detailed cavitation study will be reported for these materials in another article,\textsuperscript{[14]} we examine in the present article the tensile creep rupture data for both the preceding experiments and new experiments on DSC-Al. Their creep rupture, as well as that for particulate- and dispersion-strengthened aluminum from the literature, is discussed in the light of the Monkman–Grant relationships (Eqs. [1] and [2]).

II. EXPERIMENTAL PROCEDURE

As described in more detail in earlier publications,\textsuperscript{[13,15]} the materials used here were cast DSC-Al supplied by Chesapeake Composite Corp. (New Castle, DE), consisting of a 99.9 pct pure aluminum matrix and about 25 vol pct of 99.8 pct pure α-Al\textsubscript{2}O\textsubscript{3} particles with mean diameter 0.28 ± 0.03 μm. The as-cast billets exhibited very large grains (above 1 mm), while billets extruded at 550 °C had a recrystallized structure and showed equiaxed grains about 1.3 μm in diameter. Because of pinning by dispersoids, these grains did not grow at the testing temperature.\textsuperscript{[13]} Furthermore, a strong (111) texture was observed in the extrusion direction, as also reported for other extruded dispersion-strengthened aluminum materials.\textsuperscript{[15]}

Cylindrical dog-bone creep specimens with 4.06-mm gage diameter and 25.40-mm gage length were machined with their axes in the main billet or extrusion direction. Uniaxial tensile experiments were performed under constant-load conditions in air at temperatures between 335 °C and 500 °C. Strain was measured with an extensometer connected to a linear voltage displacement transducer (resolution ± 2.5 μm) and temperature was measured by two thermocouples on the gage length. Throughout the experiments, temperature was constant within 1 K of the nominal value. Several creep experiments were interrupted for density measurements at regular strain intervals until failure occurred, as described in more detail in another article.\textsuperscript{[14]}

III. EXPERIMENTAL RESULTS

Figure 1 shows a typical creep curve of DSC-Al with the three stages of primary, secondary, and tertiary creep strain. The secondary strain ε\textsubscript{s} is determined from the difference between the intersection of the tangent at the minimum slope of the creep curve (minimum creep rate ˙ε\textsubscript{min}) with the strain axis at t = 0 and at t = t\textsubscript{f}, so that ε\textsubscript{s} = t\textsubscript{f} ˙ε\textsubscript{min}. The primary strain ε\textsubscript{p} is given by the intersection of the tangent at the minimum slope of creep curve with the strain axis at t = 0. The tertiary strain ε\textsubscript{t} is the difference between the failure strain ε\textsubscript{f} and the sum of primary and secondary strains ε\textsubscript{p} + ε\textsubscript{s}.

Figures 2(a) and (b) give for as-cast and extruded DSC-Al semilogarithmic plots of the failure strain ε\textsubscript{f}, the secondary strain ε\textsubscript{s}, and the relative strain per creep stage ε\textsubscript{i}/ε\textsubscript{f} (i = 1, 2, 3) as a function of the minimum creep rate ˙ε\textsubscript{min}. These figures show that the total strain and secondary strain tend to increase with increasing creep rate. While there is considerable experimental scatter, Figures 2(a) and (b) also show that the secondary strain remains a relatively constant fraction of the failure strain (about 50 pct), while the tertiary contribution to strain increases at the expense of the primary contribution with increasing minimum creep rate.

The failure strain, the secondary strain, and the fraction of failure strain accumulated in each creep stage for as-cast and extruded DSC-Al are shown as a function of stress and temperature in Figures 3 and 4, respectively. At constant stress (Figure 3), the failure strain and the secondary strain increase with increasing stress, while the ratio of the secondary strain to the failure strain remains at about 50 pct within experimental errors. At a constant stress (Figure 4), the same trends are observed with increasing temperature. These trends are expected from Figure 2, because strain rate increases with increasing stress and temperature.

As demonstrated through density measurements,\textsuperscript{[13,14]} significant cavitation occurs in the secondary creep regime, where a pseudo-steady-state is established. Figures 5(a) and (b) shows typical scanning electron micrographs of fractured surfaces of creep DSC samples. The cast, coarse-grained material shows individual dimples (Figure 5(a)) containing dispersoids, while the extruded, fine-grained material (Figure 5(b)) exhibits many particles at the fracture surface, however, without well-formed dimples. Thus, in both cases, cavities most probably nucleated near the dispersoids and coalesced to result in macroscopic fracture.
Fig. 2—Total strain $\varepsilon_t$, secondary strain $\varepsilon_s$, and relative strain for each of the three creep stages $\varepsilon_t/\varepsilon_s$ as a function of minimum creep rate for (a) cast DSC-Al and (b) extruded DSC-Al.

All creep data are tabulated in Table I, which summarizes both uninterrupted experiments (for which only minimum creep rates were reported in an earlier article) and new experiments where creep was interrupted at regular strain intervals (typically 0.5 to 1 pet) for density measurements, as described in more detail in another article.

By varying the constant $K$ in the Larson–Miller parameter (Eq. [3]) between 10 and 30, the best overall fit for all DSC-Al specimens was found for $K = 18$. As shown in Figure 6(a), the Larson–Miller parameter then predicts reasonably well the failure time from the applied stress and temperature. However, the as-cast data are somewhat offset from the extruded data, indicating that, for a given stress and temperature, the failure time is slightly smaller for the cast material than for the extruded material.

As reported in an earlier article, the apparent activation energy for cast and extruded DSC-Al increases from 150 to 450 kJ/mol with increasing applied stress. By varying within these bounds the value of the activation energy in the Orr–Sherby–Dorn parameter (Eq. [4]) by intervals of 25 kJ/mol, the best overall fit for all DSC-Al data was found with $Q = 250$ kJ/mol. As shown in Figure 6(b), the Orr–Sherby–Dorn parameter also predicts reasonably well the failure time from the applied stress and temperature.

According to Eqs. [1] and [2], the minimum creep rate is plotted against the failure time in Figure 7(a) and against the failure time divided by the failure strain time in Figure 7(b). While, for a given creep rate, the failure time is similar at high creep rates ($\dot{\varepsilon} > 10^{-6}$) for both coarse-grained, cast DSC-Al and fine-grained, extruded DSC-Al, the failure time is slightly smaller at lower creep rates for the former materials (Figure 7(a)). This difference between materials disappears in the modified Monkman–Grant plots, where the failure strain is divided by the failure time (Figure 7(b)).

IV. DISCUSSION

At a given stress, the Larson–Miller parameter (respectively, Orr–Sherby–Dorn parameter) is somewhat smaller (respectively, larger) for cast DSC-Al than for extruded DSC-Al (Figures 6(a) and (b)); this indicates that the failure time at constant temperature and stress is somewhat shorter for the former material than for the latter. If cavitation at
Fig. 4—Temperature dependence of total strain $\varepsilon$, secondary strain $\varepsilon_s$, relative strain in secondary stage $\varepsilon_s/\varepsilon$, for (a) cast DSC-Al, at stresses of 56.2 and 69 MPa, and (b) extruded DSC-Al, at stresses of 41.8 and 69 MPa.

Grain boundaries was the dominant fracture mechanism, a much larger difference would be expected between cast DSC-Al (with millimeter-size grains) and extruded DSC-Al (with micrometer-size grains). Rather, Figure 6 suggests that cavitation at the dispersoid/matrix interface is controlling; the slightly earlier failure of the cast material is most likely due to pre-existing cavities formed during solidification but eliminated during extrusion. The offset between the as-cast and extruded materials disappears at 450 °C, indicating that cavity nucleation and growth are rapid enough in both types of material such that failure is not significantly influenced by pre-existing cavities in the cast material.

The Monkman–Grant plots (Figures 7(a) and (b)) show that there is very little difference between interrupted and uninterrupted creep tests. The Monkman–Grant parameters (exponents $m$ and $m'$ and constants $C$ and $C'$ in Eqs. [1] and [2]) as determined from best-fit lines with and without interrupted tests are listed in Table II. No significant effect is observed for the exponents, while the constants are somewhat larger when excluding interrupted tests from the cast data, indicating that these tests tend to have somewhat smaller failure time or ratios of failure strain to failure time. No effect was observed for the extruded specimens, possibly because only two interrupted experiments were carried out.

Figure 8(a) compares the Monkman–Grant relationships for unalloyed aluminum of varying purity (99.995, 99.3, and 98.2 pct)\textsuperscript{[17]} aluminum containing less than 2 vol pct of fine aluminum carbide and oxide produced by mechanical alloying (MA-Al),\textsuperscript{[18]} unalloyed aluminum matrix composites containing 10, 20, and 30 vol pct of 1.7 μm SiC particles (PM-Al/SiC),\textsuperscript{[19]} and coarse and fine-grained DSC-Al. The Monkman–Grant coefficients are listed in Table III, together with the material microstructural characteristics. The Monkman–Grant exponents $m = 0.838$ and $m = 0.931$ (Table III) for the large- and fine-grained DSC-Al, respectively, are very close to the exponent $m = 0.895$ for unalloyed aluminum.\textsuperscript{[17]} The constants $C = 0.134$ and 0.050 for the large- and fine-grained DSC-Al, however, are much lower than $C = 1.044$ for aluminum.\textsuperscript{[17]} This indicates that for a given creep rate, the introduction of 25 pct alumina dispersoids reduces the failure time by about one and a half orders of magnitude. However, for a given strain rate, the creep stress or temperature is much higher for DSC-Al.
### Table 1: Tensile Creep Data for DSC-Al Materials

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Note: Data from interrupted creep tests for density measurements are marked with a single asterisk for DSC-Al with 25 pct Al2O3 and with a double asterisk for DSC-Al with 33 pct Al2O3. Data in parentheses are from prematurely fractured samples and were not used in best fits or figures.

than for pure aluminum, reflecting the strengthening effect of the alumina dispersoids in the former material.

In Figure 8(a), curves for PM-Al/2SiC composites with 10 vol pct SiC particles 1.7 µm in diameter[18] overlap the DSC-Al curves, while curves for PM-Al/2SiC with 20 and 30 vol pct SiC are shifted to lower failure times by a factor of 3 to 4. The failure times of MA-Al materials (fine-grained AICO and coarse-grained AlCOR[16]) are also shorter than for DSC-Al but similar to the PM-Al/20-30 SiC curves, despite the difference in specific interfacial area and thus in nucleation sites.

Many materials show an exponent \(m'\) near unity[21] for the modified Monkman–Grant equation (Eq. [2]). As shown in Figure 8(b), data for both large-grained and fine-grained DSC-Al are well fitted by this relationship with exponents very close to unity (\(m' = 0.993\) and 1.018, respectively, Table II). Figure 8(b) also shows that DSC-Al, MA-Al[16] and PM-Al/2SiC[18] all overlap with the line for unreinforced aluminum,[17] despite significant differences in individual values of temperature, stress, creep rate, failure time, failure strain, and microstructural parameters. These parameters (matrix purity, matrix grain size \(D\), particle diameter \(d\), and particle volume fraction \(f\)) are also given in Table III, from which the specific particle-matrix interfacial area (defined as the area of particles per unit volume) can be calculated as

\[
\alpha_p = \frac{6f}{d}
\]  

assuming spherical particles arranged in a simple cubic packing. The specific matrix grain boundary area (defined as the area of grain boundaries per unit volume) is

\[
\alpha_g = \frac{3}{D}
\]  

assuming cubic grains. The total specific boundary area is then

\[
\alpha_{\text{tot}} = \alpha_p + \alpha_g
\]  

assuming that no particles are at grain boundaries, e.g., in cast specimens. If all particles are at grain boundaries (e.g., after recrystallization when particles pin grain boundaries), the total specific boundary area is

\[
\alpha_{\text{tot}} = \alpha_p + (\alpha_g - \alpha_p/4)
\]
where the second term in parentheses (set to zero when negative) is the boundary contribution, which contains a correction factor \( \alpha_s/4 \) taking into account the reduced grain boundary area due to the presence of spherical particles at grain boundaries.

If cavitation occurs at both grain boundaries and reinforcement-matrix interfaces, the total specific boundary area \( \alpha_{tot} \) is expected to have an effect on the constant \( C \) in Eq. [1]: Figure 9 indeed shows that \( C \) decreases with increasing \( \alpha_{tot} \). Since the exponents \( m \) are different among these materials, the significance of this observation is limited to large creep rates near \( \dot{\varepsilon}_{min} = 1 \text{ s}^{-1} \): Figure 9 then suggests that a larger total specific boundary area leads to lower failure times, provided the data of Figure 8(a) (which extends over about six orders of magnitude from \( \dot{\varepsilon}_{min} = 10^{-9} \text{ s}^{-1} \) to \( \dot{\varepsilon}_{min} = 10^{-3} \text{ s}^{-1} \)) can be extrapolated by three orders of magnitude to \( \dot{\varepsilon}_{min} = 1 \text{ s}^{-1} \). This prediction seems intuitively correct, since a larger boundary area provides more nucleation sites for cavities.

![Graphs and data plots](attachment:image.png)

Fig. 6—(a) Larson–Miller parameter (Eq. [3]) and (b) Orr–Sherby–Dorn parameter (Eq. [4]) for cast DSC-Al (open symbols) and extruded DSC-Al (solid symbols). Crossed symbols are for periodically interrupted creep tests. Temperature \( T \) and failure time \( t_f \) are expressed in Kelvin and hours, respectively.

![Graphs and data plots](attachment:image.png)

Fig. 7—Creep fracture data of DSC-Al plotted according to (a) the Monkman–Grant relationship (Eq. [1]) and (b) the modified Monkman–Grant relationship (Eq. [2]). Crossed symbols are for periodically interrupted creep tests, and prematurely fractured samples (in parentheses in Table I) are not included in plots.

Table II. Coefficients of the Monkman–Grant equation \( (C \) and \( m, \) Eq. [1]) and the modified Monkman–Grant equation \( (C' \) and \( m', \) Eq. [2]) for DSC-Al

<table>
<thead>
<tr>
<th>Material</th>
<th>( m )</th>
<th>( C )</th>
<th>( m' )</th>
<th>( C' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large-grained</td>
<td>0.838</td>
<td>0.134</td>
<td>0.993</td>
<td>0.515</td>
</tr>
<tr>
<td></td>
<td>(0.833)</td>
<td>(0.147)</td>
<td>(0.993)</td>
<td>(0.552)</td>
</tr>
<tr>
<td>Fine-grained</td>
<td>0.931</td>
<td>0.050</td>
<td>1.018</td>
<td>0.416</td>
</tr>
<tr>
<td></td>
<td>(0.932)</td>
<td>(0.051)</td>
<td>(1.019)</td>
<td>(0.422)</td>
</tr>
</tbody>
</table>

**Values in parentheses are derived from the best-fit lines without data from interrupted creep tests.**

of creep cavitation have been advanced to explain the physical origin of the Monkman–Grant relation, as briefly summarized in Section I. If all creep voids are assumed to nucleate on grain boundaries at the nucleation time \( t_n \) and if unconstrained void growth is controlled by power-law creep alone, the product \( t_n \cdot \dot{\varepsilon}_{min} \) for constant-load experiments is given by:

\[ t_n \cdot \dot{\varepsilon}_{min} = \frac{1}{V_n} \]

where \( V_n \) is the volume of the initial void.
Fig. 8—Comparison of (a) the Monkman–Grant relationship (Eq. [1]) and (b) the modified Monkman–Grant relationship (Eq. [2]) for Al, MA-AL, PM-Al/SiC, and DSC-Al. Materials are described in Table III and symbols are as in Fig. 7.

\[
t_f \cdot \dot{\varepsilon}_{\text{min}} \quad = \quad [(n + 1)f]^{m/(n+1)} \cdot \dot{\varepsilon}_{\text{min}} + \frac{1 - [(n + 1)f]^{m/(n+1)}}{n} \quad [9]
\]

where \( f \) is the initial area fraction of porosity and \( n \) is the stress exponent. When \( f \) goes to zero, Eq. [9] reduces to Hoff's result\(^{[19]} \)

\[
t_f \cdot \dot{\varepsilon}_{\text{min}} \quad = \quad n^{-1} \quad [10]
\]

Thus, according to Eqs. [1] and [10], the constant \( C \) in Eq. [1] determined with \( m = 1 \) should be inversely proportional to the stress exponent.

Table IV lists for pure Al, MA-AL, PM-Al/SiC composites and DSC-Al the stress exponents in compression and tension and the constants \( C \) in Eq. [1] determined for \( m = 1 \) and \( C' \) in Eq. [2] for \( m' = 1 \) (referred to as \( C_1 \) and \( C_1' \), respectively, in the following discussion). For pure aluminum, the stress exponents in tension and in compression are the same. For reinforced aluminum, the apparent stress exponent \( n \) is higher than the stress exponent \( n' \) for the pure matrix because of the presence of a threshold stress (as described in the Appendix). Furthermore, the different values of apparent stress exponents in tension and compression can be explained by a change of threshold stress due to a change of dominant deformation mechanism (e.g., from dislocation creep in compression to cavitation creep in tension for DSC-Al\(^{[18]} \)). The constant \( C_1 \) is plotted as a function of the stress exponent in Figure 10, which shows a trend in general agreement with Eq. [10], especially for the stress exponents in compression, which represent the intrinsic flow behavior of the materials. Figure 10 also shows Eq. [9] plotted for reasonable values of initial area fraction of porosity (\( f_i = 0.1 \) and 1 pct) and assuming that nucleation

<p>| Table III. Material Parameters (Matrix Grain Size ( D ), Particle Volume Fraction ( f ), and Particle Diameter ( d ), Creep Temperature ( T ), Creep Stress ( \sigma ), and Monkman–Grant Coefficients ( C ) and ( m ) in Eq. [1] and ( C' ) and ( m' ) in Eq. [2]), Derived for Failure Time and Minimum Creep Rate in Units of Seconds and Reciprocal Seconds) for Pure Al, MA-AL, PM-Al/SiC, and DSC-Al |</p>
<table>
<thead>
<tr>
<th>Materials</th>
<th>Matrix (Pct Al)</th>
<th>( D (\mu m) )</th>
<th>( f (Vol Pct) )</th>
<th>( d (\mu m) )</th>
<th>( T (\degree C) )</th>
<th>( \sigma (MPa) )</th>
<th>( m )</th>
<th>( C )</th>
<th>( m' )</th>
<th>( C' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(^{[1]})</td>
<td>—</td>
<td>—</td>
<td>0</td>
<td>—</td>
<td>260 to 593</td>
<td>—</td>
<td>0.85</td>
<td>1.360</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>A(^{[17]})</td>
<td>—</td>
<td>—</td>
<td>0</td>
<td>—</td>
<td>370 to 593</td>
<td>6 to 87</td>
<td>0.85*</td>
<td>19.953*</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>99.95</td>
<td>1000</td>
<td>370 to 593</td>
<td>18 to 177</td>
<td>0.895</td>
<td>1.044</td>
<td>0.987</td>
<td>0.751</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>99.3</td>
<td>20 to 30</td>
<td>482</td>
<td>67 to 179</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>98.2</td>
<td>50 to 100</td>
<td>482</td>
<td>67 to 179</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MA-AL(^{[16]})</td>
<td>pure Al</td>
<td>0.11 Al(_2)C(_3)</td>
<td>0.03</td>
<td>500</td>
<td>18 to 35</td>
<td>0.736</td>
<td>0.201</td>
<td>0.962</td>
<td>0.316</td>
<td></td>
</tr>
<tr>
<td>ICO</td>
<td>1.9</td>
<td>+</td>
<td>400, 500</td>
<td>26 to 38</td>
<td>0.641</td>
<td>0.542</td>
<td>0.934</td>
<td>0.796</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AICOR</td>
<td>1.75 Al(_2)O(_3)</td>
<td>500</td>
<td>26 to 38</td>
<td>0.735</td>
<td>0.423</td>
<td>0.996</td>
<td>0.397</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PM-Al/SiC(^{[18]})</td>
<td>99.6</td>
<td>330</td>
<td>1.7</td>
<td>350</td>
<td>0.736</td>
<td>0.201</td>
<td>0.962</td>
<td>0.316</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 SiC</td>
<td>25 to 31</td>
<td>0.738</td>
<td>0.846</td>
<td>1.110</td>
<td>0.040</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 SiC</td>
<td>36 to 44</td>
<td>0.792**</td>
<td>0.999**</td>
<td>0.911**</td>
<td>2.177**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30 SiC</td>
<td>49 to 54</td>
<td>0.792**</td>
<td>0.999**</td>
<td>0.911**</td>
<td>2.177**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DSC-Al</td>
<td>99.9</td>
<td>—</td>
<td>—</td>
<td>1000</td>
<td>335 to 450</td>
<td>30 to 100</td>
<td>0.838</td>
<td>0.134</td>
<td>0.993</td>
<td>0.515</td>
</tr>
<tr>
<td>cast</td>
<td>—</td>
<td>350 to 500</td>
<td>42 to 100</td>
<td>0.391</td>
<td>0.050</td>
<td>1.018</td>
<td>0.416</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>extruded</td>
<td>1.3</td>
<td>—</td>
<td>—</td>
<td>1000</td>
<td>335 to 450</td>
<td>30 to 100</td>
<td>0.838</td>
<td>0.134</td>
<td>0.993</td>
<td>0.515</td>
</tr>
</tbody>
</table>

\* Determined using units of hour and percent per hour, respectively.

\** Calculated using data for both Al-20 pct SiC and Al 30 pct SiC.
Fig. 9—Constant $C_i$ (Eq. [1]) as a function of the total specific boundary area $\alpha_{mag}$ (Eq. [8]) for materials in Table III and Figure 8. Full and empty symbols are for coarse- and fine-grained materials, respectively.

is instantaneous ($t_i = 0$). Finally, we note that the similarity between Figures 9 and 10 is not fortuitous. As derived in the Appendix (Eq. [A9]), the apparent stress exponent $n$ can be related to the specific particle-matrix interfacial area $\alpha_{mag}$, explaining the similarity between Figures 9 and 10.

We now consider the constants in the Monkman–Grant relationships. First, for $C$ in the modified Monkman–Grant relationship, Eq. [2] can be expressed as

$$t_f \cdot \dot{\varepsilon}_{\text{min}} = C_i' \cdot \varepsilon_f$$  \hspace{1cm} [11]

if $m'$ is assumed to be unity. Since by definition $t_f \cdot \dot{\varepsilon}_{\text{min}} = \varepsilon_f$, Eq. [11] can be rewritten as

$$\varepsilon_f = C_i' \cdot \varepsilon_f$$  \hspace{1cm} [12]

This equation shows that the secondary strain is proportional to the failure strain, with the proportionality constant $C_i'$ determined by best fit with Eq. [2] and $m' = 1$ as about 0.49 for DSC-Al (Table IV). This value is in a good agreement with experimental average values of $\dot{\varepsilon}/\varepsilon_f$, which are 0.47 for cast DSC-Al and 0.52 for extruded DSC-Al, as shown in Figure 2. Second, we consider the constant $C$ in the Monkman–Grant relationship (Eq. [1]). Combining Eqs. [2], [10], and [12] gives

$$t_f \cdot \dot{\varepsilon}_{\text{min}} = C_i' \cdot n$$  \hspace{1cm} [13]

which is equivalent to Eq. [1] with $m = m' = 1$ and thus gives an expression for $C_i'$.

**Table IV. Apparent Stress Exponent, n, Coefficients of Monkman–Grant equations $C_i$ and $C_i'$ (Determined with $m = 1$ and $m' = 1$ from Eqs. [1] and [2]) for Pure Al$^{[17]}$ MA-Al$^{[16]}$ PM-Al/AlSiC$^{[18,23]}$ and DSC-Al$^{[23]}$**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Al</th>
<th>AICO</th>
<th>MA</th>
<th>AICOR</th>
<th>Al/AlSiC</th>
<th>DSC</th>
<th>DSC-Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$ (Tension)</td>
<td>4.4</td>
<td>17</td>
<td>41</td>
<td>32</td>
<td>11</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>$n$ (Compression)</td>
<td>4.4</td>
<td>28</td>
<td>48</td>
<td>18</td>
<td>22</td>
<td>23</td>
<td>23</td>
</tr>
<tr>
<td>$C_i$</td>
<td>0.383</td>
<td>0.042</td>
<td>0.016</td>
<td>0.007</td>
<td>0.033</td>
<td>0.034</td>
<td>0.034</td>
</tr>
<tr>
<td>$C'_i$</td>
<td>0.619</td>
<td>0.361</td>
<td>0.425</td>
<td>0.306</td>
<td>0.500</td>
<td>0.481</td>
<td>0.481</td>
</tr>
</tbody>
</table>

Fig. 10—Constant $C_i$ (determined with $m = 1$ from Eq. [1]) as a function of the apparent stress exponent for materials in Table III and Fig. 8.

Thus, the preceding paragraph shows that, if a material exhibits a secondary creep strain proportional to its failure strain, the modified Monkman–Grant relationship (Eq. [2]) with $m' = 1$ follows, in good agreement with experiments. Furthermore, if Hoff’s equation holds (Eq. [10]), the Monkman–Grant relationship (Eq. [1]) with $m = 1$ can be derived. The experimental observation that $m$ is often less than unity, however, cannot be explained at the present time and may be due to other effects not taken into account in the derivation, e.g., primary creep deformation.

**V. CONCLUSIONS**

The creep fracture of coarse- and fine-grained DSC-Al with 25 vol pct of 0.3 $\mu$m Al$_2$O$_3$ dispersoids was investigated for temperatures between 335 °C and 500 °C and tensile stresses between 30 and 100 MPa. The following conclusions were reached.

1. Creep strains for primary, secondary, and tertiary creep stages can be correlated to the minimum creep rate. The total failure strain and the secondary strain increase with increasing minimum creep rate, or equivalently with increasing stress at constant temperature and with increasing temperature at constant stress.

2. The fracture surface of crept samples indicates that cavities nucleate at particle-matrix interfaces, not at grain boundaries. This is supported by the observation that as-cast and extruded DSC-Al have similar failure strain and time, despite a difference of six orders of magnitude in specific grain boundary area.

3. The creep stress of DSC-Al can be reasonably well correlated with a Larson–Miller parameter or an Orr–Sherby–Dorn parameter, which combines temperature and failure time. Failure time of DSC-Al can thus be interpolated from experimental stress and temperature.

4. The minimum creep rate of DSC-Al correlates very well with the failure time (Monkman–Grant relation) and the failure time divided by the failure strain (modified Monkman–Grant relation). Good fits are also found for literature data on dispersion-strengthened aluminum and aluminum matrix composites spanning a large range of...
The constants $C_1$ and $C_1'$ in the Monkman–Grant relations (determined for Monkman–Grant exponents of unity) decrease with increasing apparent stress exponent, as predicted by Cocks and Ashby’s cavitation model and Hoff’s viscoelastic creep theory. The apparent stress exponent is shown to be proportional to the specific particle–matrix interfacial area for the preceding materials.

For DSC-AI, the ratio between secondary strain and failure strain is constant. Then, both the original and modified Monkman–Grant relations with exponents of unity can be derived, in agreement with experimental data.

ACKNOWLEDGMENTS

This research was supported by the National Science Foundation under Grant No. DMR 9417636, Dr. Bruce McDonald, monitor. The authors also acknowledge Chesapeake Composite Corp. for supplying DSC materials.

Appendix: Derivation of the relationship between the specific interfacial parameter $\sigma_s$ and the apparent stress exponent $n$

The minimum creep rate $\dot{\varepsilon}$ of composites and dispersion-strengthened metals can be expressed as a power law:

$$\dot{\varepsilon} = A\sigma^n \exp\left(\frac{-Q}{RT}\right)$$  \hspace{1cm} [A1]

where $\sigma$ is the applied tensile stress, $n$ the apparent stress exponent, $Q$ the apparent activation energy, $R$ the gas constant, $T$ the absolute temperature, and $A$ a function of the matrix shear modulus and temperature. If a threshold stress $\sigma_{th}$ is considered, Eq. [A1] can be rewritten as

$$\dot{\varepsilon} = A'(\sigma - \sigma_{th})^n \exp\left(\frac{-Q}{RT}\right)$$  \hspace{1cm} [A2]

where $A'$, $n'$, and $Q'$ are the power-law constant, stress exponent, and activation energy of the unstrengthened matrix, respectively. From Eqs. [A1] and [A2], the apparent stress exponent $n$ can then be expressed as

$$n = n' \cdot \frac{\sigma}{\sigma - \sigma_{th}}$$  \hspace{1cm} [A3]

at constant threshold stress and temperature. Equation [A3] can be rewritten as

$$n = n' \cdot \left(1 + \frac{\sigma_{th}}{\sigma - \sigma_{th}}\right)$$  \hspace{1cm} [A4]

Thus, the apparent stress exponent $n$ increases with increasing threshold stress $\sigma_{th}$ for a constant effective stress ($\sigma - \sigma_{th}$).

The threshold stress for power-law creep in Eq. [A4] can be expressed as a fraction of the Orowan stress $\sigma_{or}$ as

$$\sigma_{th} = \beta \cdot \sigma_{or}$$  \hspace{1cm} [A5]

where $\beta$ can take values between 0 and 1, depending on the dominant threshold mechanism. The Orowan stress in Eq. [A5] is given by

$$\sigma_{or} = M \cdot \frac{0.4 \cdot G \cdot b}{\pi \cdot d} \cdot \frac{\ln(\tilde{d}/b)}{\sqrt{1 - \nu}}$$  \hspace{1cm} [A6]

where $M$ is the matrix mean orientation factor, $G$ its shear modulus, $\nu$ its Poisson’s ratio, and $b$ its Burger’s vector; where $\tilde{d} = \sqrt{2/3d}$ is the mean diameter of a circular section in a random plane for a spherical particle of diameter $d$; and where $\lambda$ is the mean interparticle distance:

$$\lambda = \tilde{d} (\sqrt{\pi/4f} - 1)$$  \hspace{1cm} [A7]

assuming a square arrangement of spherical dispersoids of volume fraction $f$.

The inverse of $\lambda$ in the expression in parentheses in Eq. [A7] can be reasonably well approximated as

$$(\sqrt{\pi/4f} - 1)^{-1} \approx 6 \cdot f$$  \hspace{1cm} [A8]

for $0 < f < 0.4$, as shown in Figure A1.


$$n = n' + B \cdot \ln \left(\frac{\tilde{d}/b}{\sigma_{or}}\right)$$  \hspace{1cm} [A9]

where the constant $B$ is given by

$$B = \frac{\sqrt{6} \cdot \beta \cdot M}{5 \cdot \pi \cdot \sqrt{1 - \nu}} + \frac{n' \cdot G \cdot b}{\sigma - \sigma_{th}}$$  \hspace{1cm} [A10]

and contains only materials parameters and the effective stress ($\sigma - \sigma_{th}$) assumed to be constant. Equation [A9] thus shows that the apparent stress exponent is linearly related to the specific particle–matrix interfacial area, provided the particle diameter and the effective stress are constant (this condition is not fulfilled for the materials in Table III showing a wide range of particle sizes).

REFERENCES