A UNIFIED INTERPRETATION OF THRESHOLD STRESSES IN THE CREEP AND HIGH STRAIN RATE SUPERPLASTICITY OF METAL MATRIX COMPOSITES

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(Received 4 May 1999; accepted 28 June 1999)

Abstract—The flow behavior of metal matrix composites is characterized by the presence of a threshold stress under both creep conditions at intermediate temperatures and in high strain rate superplasticity (HSR SP) at very high temperatures near the onset of partial melting. Experiments show the measured threshold stresses decrease with increasing temperature and this trend has been interpreted using an Arrhenius-type relationship incorporating an energy term, $Q_0$. Typically, the experimental values reported for $Q_0$ are ~20–30 kJ/mol under creep conditions but up to ~100 kJ/mol in experiments associated with HSR SP. This report resolves this apparent dichotomy by demonstrating that both sets of results become consistent when the analysis is extended to incorporate an additional dependence on temperature associated with load transfer and substructure strengthening.

Keywords: Composites; Creep; High temperature mechanical properties; Superplasticity

1. INTRODUCTION

The steady-state creep rate, $\dot{\varepsilon}$, of pure metals and simple alloys generally depends upon stress and temperature through a relationship of the form

$$\dot{\varepsilon} = \frac{ADG^b}{kT} \left( \frac{\sigma}{G} \right)^n$$  \hspace{1cm} (1)

where $D$ is the diffusion coefficient [equal to $D_0 \exp(-Q/RT)$], where $D_0$ is a frequency factor, $Q$ is the true activation energy for creep, $R$ is the gas constant and $T$ is the absolute temperature, $G$ is the shear modulus, $b$ is the Burgers vector, $k$ is Boltzmann’s constant, $\sigma$ is the applied stress, $n$ is the true stress exponent and $A$ is a dimensionless constant. For these materials, a logarithmic plot of $\dot{\varepsilon}$ against $\sigma$ gives datum points lying along a series of straight lines for each separate testing temperature provided a single mechanism is rate-controlling over the experimental range of stress and temperature.

For more complex materials, where obstacles are present within the microstructure which impede the movement of dislocations, logarithmic plots of $\dot{\varepsilon}$ against $\sigma$ exhibit significant curvature so that the apparent stress exponent, $n_a$, reaches very high values at the lowest levels of the applied stress. It is a standard procedure in these materials to interpret the deformation in terms of an effective stress, $\sigma_e$, which is defined as $(\sigma - \sigma_0)$ where $\sigma_0$ is a threshold stress delineating a lower limiting stress for any measurable flow [1]. Under these conditions, equation (1) is replaced by

$$\dot{\varepsilon} = \frac{ADG^b}{kT} \left( \frac{\sigma - \sigma_0}{G} \right)^n.$$  \hspace{1cm} (2)

The high temperature flow behavior of metal matrix composites (MMCs) is generally interpreted using equation (2) when data are collected over a sufficiently wide range of strain rates to reveal the curvature in the logarithmic plots of $\dot{\varepsilon}$ vs $\sigma$. The values of the threshold stresses, $\sigma_0$, are then estimated either by plotting $\dot{\varepsilon}^{1/n}$ against $\sigma$ on linear axes for selected values of $n$ and extrapolating linearly to zero creep rate [2] or by directly extrapolating the creep data to very low strain rates [3].

Threshold stresses have been estimated from tests conducted on a number of MMCs and the results reveal that the values of $\sigma_0$ generally decrease with increasing temperature. This temperature dependence is not fully understood at the present time but it is consistent with the predictions from recent theoretical models for the creep of dispersion strengthened alloys [4, 5]. It has been proposed that it may be appropriate to express the temperature dependence of the normalized threshold stress,
\[ \frac{\sigma_0}{G} = B \exp \left( \frac{Q_0}{RT} \right) \]  

(3)

where \( B \) is a constant and \( Q_0 \) is an energy term which appears to be associated with the process by which the mobile dislocations overcome the obstacles in their glide planes.

Equation (3) has been used to analyze the mechanical properties of MMCs at elevated temperatures under two different experimental conditions.

First, this approach is used widely under high temperature creep conditions where, typically, the materials are tested at temperatures in the vicinity of \( \sim 0.6T_m \), where \( T_m \) is the absolute melting temperature. For a series of Al-based MMCs tested in creep, a tabulation shows that the values derived for \( Q_0 \) in equation (3) lie consistently within the rather narrow range of \( \sim 20–30 \) kJ/mol and very similar values of \( Q_0 \) are obtained also using the same analysis for the creep of unreinforced aluminum alloys fabricated using either powder metallurgy or rapid solidification procedures [7]. It is instructive to note these values for \( Q_0 \) are similar to the binding energies between dislocations and impurity atoms [8] and this is consistent with experimental observations demonstrating direct interactions between dislocations and dispersed particles or precipitates in the glide planes [7, 9]. Second, equation (3) has been used to analyze the flow behavior of MMCs exhibiting high strain rate superplasticity (HSR SP) where the materials are tested at very high temperatures, typically in the vicinity of \( \sim 0.9T_m \). At these high temperatures, however, \( \sigma_0 \) generally exhibits a very strong dependence on temperature [10–13] so that the values estimated for \( Q_0 \) are exceptionally high: for example, \( Q_0 \approx 94 \) kJ/mol for HSR SP in a composite having an Al-2124 matrix reinforced with SiC whiskers [14]. Thus, although data have been analyzed from both creep and HSR SP using the concept of an effective stress and equation (3), there is a very clear dichotomy between the low values of \( Q_0 \) reported for the creep of MMCs at the lower temperatures and the high values of \( Q_0 \) reported for HSR SP in MMCs at very high testing temperatures.

The present work was motivated by this apparent disagreement. As will be demonstrated, both sets of data become consistent when it is recognized that it is necessary to incorporate into the analysis an additional dependence on temperature due to load transfer and substructure strengthening.

2. FACTORS INFLUENCING THE MEASURED CREEP RATES IN MMCs

It is now generally accepted that no significant plastic flow occurs within the rigid ceramic reinforcements which are incorporated into the MMCs so that plastic deformation is controlled exclusively by flow in the matrix materials [7, 15–17]. This conclusion is supported by experimental evidence showing similar relationships between \( \dot{\varepsilon} \) and \( \sigma \), including the presence of threshold stresses, in both MMCs and their unreinforced matrix alloys: for example, in a powder metallurgy (PM) Al-6061 alloy and a PM Al-6061–30 vol.% SiC(p) composite [18] and in a PM Al-6092 alloy and a PM Al-6092–25 vol.% SiC(p) composite [19], where \( p \) denotes particulate reinforcement. Despite this similarity, however, the creep rates recorded in the matrix alloys are consistently faster than in the MMCs and this difference may be revealed by logarithmically plotting the creep data for the two materials either as \( \dot{\varepsilon} \) vs \( (\sigma - \sigma_0) \) at the same testing temperature [19] or in the normalized form of \( \dot{\varepsilon}kT/DGb \) vs \( (\sigma - \sigma_0)/G \) for more than one testing temperature [17]. The latter format is illustrated schematically in Fig. 1 and it is apparent that any selected strain rate necessitates the use of a higher effective stress in the MMC or, conversely, the measured strain rate is lower in the MMC for any selected level of the effective stress.

There are two possible origins for the additional creep strengthening evident in the composite.

First, the reinforcement plays a critical role in MMCs because there is the possibility that, through
the process of load transfer, part of the external load is carried by the reinforcement and there is a consequent reduction in the effective stress acting on the material [20, 21]. Recent experiments have shown that load transfer is initiated during the very early stages of deformation, prior to the onset of general yielding, both in Al-6061–20 vol.% SiC(p) [22] and in Al-2219–15 vol.% TiC(p) [23], and this suggests the effective stress in the presence of load transfer, \( \sigma_{\text{e}(LT)} \), may be expressed in the form

\[
\sigma_{\text{e}(LT)} = (1 - x)\sigma - \sigma_0 \tag{4}
\]

where \( x \) is the load transfer coefficient. In practice, the values of \( x \) will lie within the range from 0 to 1, where \( x = 0 \) represents an absence of any load transfer and increasing non-zero values of \( x \) signify an increasing transfer of the applied load to the reinforcement.

Second, there may be a substructural strengthening within the matrix due, for example, to an increase in the dislocation density as a consequence of the thermal mismatch between the matrix and the reinforcement or the presence of internal stresses because of resistance by the reinforcement to plastic flow in the matrix [24–29]. Although no attempt is made here to delineate the precise mechanisms responsible for substructure strengthening, it is apparent this strengthening may be incorporated into the analysis through the introduction of an effective stress, \( \sigma_{\text{e}(SS)} \), of the form

\[
\sigma_{\text{e}(SS)} = (1 - \chi)\sigma - \sigma_0 \tag{5}
\]

where \( \chi \) is a substructure strengthening coefficient having values from 0 to 1.

Thus, in the presence of both load transfer and substructure strengthening, there is an effective stress acting on the composite which is given by

\[
\sigma_{\varepsilon} = (1 - \beta)\sigma - \sigma_0 \tag{6}
\]

where \( \beta \) is the appropriate coefficient incorporating both load transfer and all creep strengthening processes.

In order to use equation (6) in the standard relationship for the steady-state creep rate, it is convenient to express the effective stress in the form

\[
\sigma_{\varepsilon} = (1 - \beta)\left(\sigma - \sigma_0^*\right) \tag{7}
\]

where \( \sigma_0^* \) is defined as an apparent threshold stress which is given by

\[
\sigma_0^* = \frac{\sigma_0}{(1 - \beta)}. \tag{8}
\]

It follows from equation (8) that \( \beta = 0 \) and \( \sigma_0^* = \sigma_0 \) in the absence of any load transfer or strengthening but in composite materials, where additional strengthening is often present, \( \beta \neq 0 \) and the value of \( \sigma_0 \) is effectively magnified by a factor of \( 1/(1 - \beta) \).

The incorporation of load transfer and substructure strengthening into the analysis through equation (7) requires that equation (2) is replaced by

\[
\dot{\varepsilon} = \frac{ADG\beta}{kT} (1 - \beta)^\eta \left[ \frac{\sigma - \sigma_0^*}{\sigma_0^*} \right]^n. \tag{9}
\]

From equation (9), it is now apparent that the threshold stresses estimated in creep or HSR SP, using either the extrapolation of \( \dot{\varepsilon}^{1/n} \) against \( \sigma \) on linear axes [2] or the direct extrapolation method [3], yield values for the apparent threshold stress, \( \sigma_0^* \), rather than the true threshold stress, \( \sigma_0 \), and these two values are equivalent only when \( \beta = 0 \) and there is no additional strengthening. For unreinforced materials, analyses of experimental creep data will lead to values for the true threshold stress, \( \sigma_0 \), and the correct values for \( Q_0 \). For MMCs, however, the values estimated for \( Q_0 \) depend critically upon the nature of any additional dependence on temperature which may be introduced through the strengthening factor \( \beta \). This temperature dependence may arise through a change in the mechanism of flow within the matrix or if, for example, the magnitudes of the internal stresses decrease with increasing temperature.

3. THE ADDITIONAL TEMPERATURE DEPENDENCE INTRODUCED INTO MMCs

In order to investigate the possibility of an additional dependence on temperature introduced into MMCs through the factor \( \beta \), it is necessary to make a direct comparison between the mechanical properties of the composites and their unreinforced matrix materials. In the following two sections, this comparison is undertaken separately for MMCs undergoing creep and HSR SP, respectively.

3.1. Additional temperature dependence in the creep of MMCs

Three composites were selected for examination under creep conditions: Al-2124–10 vol.% SiC(p) [15], Al–30 vol.% SiC(p) [30] and Al-6061–30 vol.% SiC(p) [31]. There were two reasons for selecting these materials. First, the creep data for each composite is very extensive, covering more than six orders of magnitude of strain rate, so that the threshold stresses are well defined. Second, creep data are available for the unreinforced Al-2124 [9] and Al-6061 [18] matrix alloys and the composite having a pure aluminum matrix may be compared with the extensive creep data summarized for pure aluminum [32].

The MMCs and their unreinforced matrix alloys deform by the same rate-controlling mechanism and therefore they have the same values for the true stress exponent, \( n \), and the true activation energy for creep, \( Q \) [7, 19]. It is reasonable to assume that, since the measured values of \( n \) and \( Q \) are inter-re-
lated in high temperature creep [33], the occurrence of load transfer and substructure strengthening will influence the measured creep rates but there will be no concomitant change in the value of the dimensionless constant \( A \) in equation (2).

Equation (9) applies also to the unreinforced matrix alloys where \( \beta = 0 \) and \( \sigma_0^* = \sigma_0 \) so that, for conditions of constant \((\sigma - \sigma_0^*)\) and at the same testing temperature, the creep rates in the composite, \( \dot{\varepsilon}_{\text{MMC}} \), and in the alloy, \( \dot{\varepsilon}_{\text{alloy}} \), are related through the expression

\[
\frac{\dot{\varepsilon}_{\text{MMC}}}{\dot{\varepsilon}_{\text{alloy}}} = (1 - \beta)^n.
\]

Thus, the value of \( \beta \) is given by

\[
\beta = 1 - \left[ \frac{\dot{\varepsilon}_{\text{MMC}}}{\dot{\varepsilon}_{\text{alloy}}} \right]^{1/n}
\]

and the experimental values of \( \beta \) may be determined by comparing the strain rates of the MMCs and the unreinforced matrix materials at a constant value of \((\sigma - \sigma_0^*)\). These comparisons are made using Fig. 2 where the strain rates are plotted logarithmically against the apparent effective stress, \((\sigma - \sigma_0^*)\).

Figure 2(a) shows a plot of \( \dot{\varepsilon} \) vs \((\sigma - \sigma_0^*)\) for Al-2124–10 vol.% SiC(p) [15] and for the unreinforced Al-2124 matrix alloy [9] at a testing temperature of 678 K. Both sets of experimental data superimpose along a single line and there is no additional strengthening in the composite so that \( \beta = 0 \). Similar results were obtained also at the other two testing temperatures of 618 and 648 K and the trend is consistent with the experimental observation that both materials exhibited similar creep rates above \( \sim 10^{-4}/s \). The absence of any additional strengthening in the composite has been attributed to the occurrence of debonding between the SiC particulates and the matrix [34].

Figure 2(b) shows a similar plot for Al–30 vol.% SiC(p) [30] at 673 K and the corresponding line predicted at this temperature for pure Al where there is no threshold stress and \( \sigma_0^* = \sigma_0 = 0 \) [32]. Inspection shows these two lines are parallel so that the true stress exponent is \( \sim 4.5 \) for both materials but the lines are not congruent and there is an additional creep strengthening in the composite which is not present in the pure metal. Similar plots were obtained for the other testing temperatures of 623 and 723 K and, using equation (11) with \( n = 4.5 \), the values of \( \beta \) were estimated for each testing temperature as documented in Table 1.

A similar plot is shown in Fig. 2(c) for Al-6061–30 vol.% SiC(p) [31] and for the unreinforced Al-6061 matrix alloy [18] at a testing temperature of 648 K. This plot also indicates the presence of some additional strengthening in the composite but with \( n = 5 \) for both materials. Using equation (11) with \( n = 5 \) gives the values for \( \beta \) shown in Table 1 for the two testing temperatures of 648 and 678 K.

The variation of \( \beta \) with temperature, \((\Delta \beta/\Delta T)\), was estimated for these MMCs and the values are included in the fourth column of Table 1: in practice, \((\Delta \beta/\Delta T)\) is less well defined for Al-6061–30 vol.% SiC(p) because there are only two testing temperatures. The value of \((\Delta \beta/\Delta T)\) is \( -1 \times 10^{-4} \) for Al–30 vol.% SiC(p) suggesting that only a minor additional dependence on temperature is introduced into the composite through the coefficient \( \beta \). This conclusion may be illustrated by plotting, on semi-logarithmic axes, the two sets of normalized threshold stresses, \( \sigma_0^*/G \) and \( \sigma_0^*/G \), against the reciprocal of the absolute temperature, \( 1/T \), as shown in Fig. 3, where \( G \) is taken as the shear modulus for pure Al \( (3.022 \times 10^4) - 16 T \) MPa [35]): this plot was constructed using the values of \( \sigma_0^* \) reported experimentally and by estimating \( \sigma_0 \) from equation (8) using the values of \( \beta \) in Table 1. Although the datum points are limited, both plots yield straight lines in agreement with equation (3). Furthermore, the values estimated for \( Q_0 \) are \( \sim 23 \) kJ/mol when using \( \sigma_0^*/G \) which neglects the temperature dependence of \( \beta \) and \( \sim 21 \) kJ/mol when using \( \sigma_0^*/G \) which includes this additional dependence on temperature. Therefore, the coefficient \( \beta \) has only a minor influence on the value of \( Q_0 \) in this material although it is apparent from Fig. 3 that the magnitude of the apparent threshold stress is higher than the true threshold stress by a factor of \( \sim 3 \) \([ \geq 1/(1 - \beta) \] at any selected temperature.

This analysis of the mechanical behavior of MMCs under creep conditions leads to an important conclusion. If \( \beta \) is independent of temperature, or if the variation of \( \beta \) with temperature is very weak as in the Al–30 vol.% SiC(p) composite, \( \sigma_0^* \) and \( \sigma_0 \) exhibit relatively similar dependences on temperature and the use of either term leads essentially to identical values for \( Q_0 \) in equation (3).

### 3.2. Additional temperature dependence in MMCs exhibiting HSR SP

Metal matrix composites exhibiting HSR SP are generally tested at very high temperatures up to and above the temperatures where partial melting may occur along the grain boundaries and/or at the reinforcement/matrix interfaces [36–38]. At temperatures below the onset of partial melting, the measured activation energies are generally higher than anticipated either for lattice self-diffusion or grain boundary diffusion in the matrix alloys [10, 13] although, by incorporating a threshold stress through equation (2), the true stress exponent is typically \( \sim 2 \) which is consistent with normal superplasticity at much lower strain rates [39].

\( ^{\dagger} \)It is important to note that the earlier experimental determinations of the threshold stresses relate strictly to \( \sigma_0^* \) rather than \( \sigma_0 \).
Fig. 2. Strain rate vs apparent effective stress for (a) Al-2124–10 vol.% SiC(p) [15] and the unreinforced Al-2124 matrix alloy [9], (b) Al–30 vol.% SiC(p) [30] and pure Al [32], and (c) Al-6061–30 vol.% SiC(p) [31] and the unreinforced Al-6061 matrix alloy [18].
Table 1. Values of the coefficient $\beta$ under creep conditions

<table>
<thead>
<tr>
<th>Material</th>
<th>$T$ (K)</th>
<th>$\beta$</th>
<th>$(\Delta \beta/\Delta T)$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al-2124–10 vol.% SiC(p) [15]</td>
<td>618</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>648</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>678</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Al–30 vol.% SiC(p) [30]</td>
<td>623</td>
<td>0.71</td>
<td>$-1 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>673</td>
<td>0.72</td>
<td></td>
</tr>
<tr>
<td></td>
<td>723</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td>Al-6061–30 vol.% SiC(p) [31]</td>
<td>648</td>
<td>0.52</td>
<td>$-1.7 \times 10^{-3}$</td>
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<td></td>
<td>678</td>
<td>0.47</td>
<td></td>
</tr>
</tbody>
</table>

In order to consider the influence of the coefficient $\beta$ on the values estimated for $Q_0$, experimental data were examined for two different Mg-based composites exhibiting HSR SP: a Mg–4% Zn alloy reinforced with 28 vol.% of Mg2Si particulates [12], designated Mg–Zn–28 vol.% Mg2Si(p), and a Mg–4% Al alloy also reinforced with 28 vol.% Mg2Si particulates [12], designated Mg–Al–28 vol.% Mg2Si(p). These two MMCs were selected for two reasons. First, they provide an opportunity to examine behavior in HSR SP both without and with the presence of any partial melting: thus, d.s.c. measurements revealed no melting up to a temperature of 823 K in Mg–Zn–28 vol.% Mg2Si(p) and this temperature is above the highest testing temperature whereas there was an onset of partial melting at 794 K in Mg–Al–28 vol.% Mg2Si(p) and these tests were conducted up to a maximum temperature of 813 K [12]. Second, an earlier analysis of HSR SP in these materials provided information on the relevant values for $\beta$ over a wide range of testing temperatures [40].†

Table 2 shows the values of $\beta$ estimated for these two composites [40] together with two additional values estimated for Mg–Al–28 vol.% Mg2Si(p) at testing temperatures above the onset of partial melting at 794 K. It is important to note that these values of $\beta$ were not obtained, as in Section 3.1, by making a direct comparison between the flow properties of the composites and the unreinforced matrix alloys because no information was available on flow in the unreinforced materials. Instead, they were obtained by making use of a phenomenological relationship derived for superplastic flow at normal strain rates from an analysis of an extensive set of experimental data [41] and then deriving a relationship in which $\beta$ was the only adjustable parameter so that the best value of $\beta$ may be determined from a line of best fit to the experimental data: a detailed description of this procedure was given earlier [40]. The fourth column of Table 2 shows the relevant values for $(\Delta \beta/\Delta T)$, with separate values indicated for Mg–Al–28 vol.% Mg2Si(p) below and above the onset of partial melting at 794 K.

Figure 4 shows the variation of the normalized threshold stresses, $\sigma_0/G$ and $\sigma_0^s/G$, with the reciprocal of the absolute temperature, $1/T$, for (a) Mg–Zn–28 vol.% Mg2Si(p) [12] and (b) Mg–Al–28 vol.% Mg2Si(p) [12], respectively, where $G$ is taken as the value for pure Mg ($1.92 \times 10^5$) – 8.67 MPa [42]: for the latter material, the onset of partial melting at 794 K is indicated by the vertical broken line. Inspection of Fig. 4(a) shows that both sets of datum points fall along straight lines but these lines have different slopes so that the values estimated for $Q_0$ are $\sim 62$ kJ/mol from $\sigma_0/G$ when the temperature dependence of $\beta$ is neglected and $\sim 33$ kJ/mol from $\sigma_0/G$ when this additional dependence on temperature is included. The former value is anomalously high and consistent with the earlier detailed analysis of Mishra et al. [14] but the latter value is close to the documented range of $\sim 20–30$ kJ/mol for $Q_0$ under creep conditions [7]. The situation is different in Fig. 4(b) because the onset of partial melting at 794 K leads to a clear deviation from linearity. The values estimated for $Q_0$ at temperatures below the onset of partial melting are $\sim 66$ kJ/mol from $\sigma_0^s/G$ and $\sim 27$ kJ/mol from $\sigma_0/G$, so that again the value of $Q_0$ is anomalously high when the temperature dependence of $\beta$ is neglected but it is within the anticipated range from creep experiments when $\sigma_0^s$ is converted to $\sigma_0$.

The datum points above the onset of partial melting in Fig. 4(b) deviate from linearity and suggest the occurrence of a different flow process when a liquid phase is present at the interfaces. This conclusion is reasonable because diffusion, and therefore the rate of deformation, is enhanced in the presence of a liquid layer [43]. Although the precise flow process under these conditions is not yet understood, it is reasonable to speculate that the diffusion coefficient, $D$, in equation (9) is replaced by an effective diffusion coefficient, $D_{eff}$, incorporating the coefficients for both grain boundary diffusion and flow in the liquid layer together with the thickness of the liquid phase. In general terms, it is anticipated that the presence of a liquid will enhance the rate of flow by displacing the experimental datum points to faster strain rates in a logarithmic plot of strain rate against stress, and this will have the effect of reducing the measured

†The earlier analysis attributed all of the additional strengthening in the composites to load transfer without including the possibility of substructure strengthening. Thus, the values reported earlier for $\beta$ [40] are equivalent to $\beta$ in the present analysis.
threshold stresses and increasing the activation energy for flow. The former trend is supported by experimental estimates of the threshold stresses in MMCs exhibiting HSR SP [40] and the latter trend is consistent with recent experimental data showing exceptionally high activation energies for flow in MMCs at temperatures immediately above the onset of partial melting [44, 45].

4. DISCUSSION

The flow behavior of metal matrix composites, when recorded over more than five orders of magnitude of strain rate and plotted logarithmically as $\dot{\varepsilon}$ against $\sigma$, typically exhibits curvature so that the apparent stress exponent increases with decreasing stress. This behavior is interpreted by invoking a threshold stress which generally is found to decrease in magnitude with increasing temperature. The dependence on temperature may be conveniently expressed through an Arrhenius-type relationship of the form shown in equation (3) which contains an energy term, $Q_0$. Experiments have shown that the values of $Q_0$ in creep lie within a narrow range of $\sim 20\text{--}30$ kJ/mol which is consistent with the binding energies between dislocations and impurity atoms whereas in HSR SP the values reported for $Q_0$ are very high and appear to be unrelated to the binding energies. The present analysis shows this apparent dichotomy may be resolved by introducing load transfer and substructure strengthening into the analysis through a strengthening coefficient, $\beta$. Thus, the variation of $\beta$ with temperature gives an additional temperature-dependent term which must be incorporated into any analysis to determine the true value of $Q_0$. In practice, creep experiments are conducted at relatively low homologous temperatures, typically of the order of $0.6T_m$ and the temperature dependence of $\beta$ is small so that there is little or no influence on the values estimated for $Q_0$; but in HSR SP the testing temperature is very high, typically $0.9T_m$, and the temperature dependence of $\beta$ is sufficiently large that it leads to anomalously high apparent values for $Q_0$ when load transfer and substructure strengthening are not included in the analysis. There are similarities in the values obtained for $Q_0$ in creep and HSR SP when the temperature dependence of $\beta$ is included in the

![Graph](image)

Fig. 3. Normalized threshold stresses vs the reciprocal of the absolute temperature for Al–30 vol.% SiC(p) [30].

<table>
<thead>
<tr>
<th>Material</th>
<th>$T$ (K)</th>
<th>$\beta$</th>
<th>$(A/\Delta T)$ (K)</th>
</tr>
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<tbody>
<tr>
<td>Mg–Zn–28 vol.% Mg2Si(p) [12]</td>
<td>653</td>
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<td></td>
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<td>733</td>
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<tr>
<td>Mg–Al–28 vol.% Mg2Si(p) [12]</td>
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<td>0.65</td>
<td>$-3.6 \times 10^{-3}$</td>
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<tr>
<td></td>
<td>753</td>
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<tr>
<td></td>
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<td></td>
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<tr>
<td></td>
<td>803</td>
<td>0.30</td>
<td>$-1.7 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>813</td>
<td>0</td>
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</table>
analysis, thereby suggesting the threshold stresses for these two testing conditions may have similar origins.

It is useful to examine the experimental factors influencing the magnitude of $\beta$. It is apparent from Table 1 that no additional strengthening is introduced into the Al-2124 alloy reinforced with 10 vol.% of SiC particulates whereas all of the other composites exhibit additional strengthening except for Mg–Al–28 vol.% Mg$_2$Si(p) above the onset of partial melting. There is an implication from this analysis that the volume fraction of the reinforcement may play a role in determining the magnitude of any additional strengthening.

In conclusion, it is important to note that load transfer and substructure strengthening are important when MMCs are tested at very high temperatures under conditions of HSR SP and it appears these processes may affect estimates of both the activation energy for flow, as documented earlier [40], and the energy term associated with the temperature dependence of the threshold stress.

5. SUMMARY AND CONCLUSIONS

1. Metal matrix composites (MMCs) exhibit flow behavior characterized by the presence of a threshold stress under conditions of both creep and high strain rate superplasticity (HSR SP).
2. The threshold stress measured experimentally is strictly equal to an apparent threshold stress, $\sigma^*_0$. 

![Diagram](image)
which is related to the true threshold stress, $\sigma_0$, through a factor of $(1 - \beta)$, where $\beta$ is a coefficient having values lying within the range from 0 to 1.

3. The dependence of the threshold stress on temperature may be expressed through an Arrhenius-type relationship incorporating an energy term, $Q_b$. When the influence of $Q_b$ is not considered, the values of $Q_b$ lie typically within the range of ~20–30 kJ/mol for MMCs under creep conditions but much higher values are reported under conditions of HSR SP.

4. By incorporating the temperature dependence of $\beta$ into the analysis, it is demonstrated that the values of $Q_b$ in HSR SP are reduced to values similar to those obtained in MMCs tested under creep conditions. This similarity suggests that the threshold stresses under these two conditions may have similar origins.

Acknowledgements—This work was supported by the National Science Foundation under Grant No. DMR-9625969 and by the U.S. Army Research Office under Grant DAAH04-96-1-0332.

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